NS – 308

III Semester B.A./B.Sc. Examination, November/December 2016
(Semester Scheme) (NS) (2012-13 and onwards)
(Repeaters – Prior to 2015-16)
MATHEMATICS – III

Time : 3 Hours Max. Marks : 100

Instruction : Answer all questions.

I. Answer any fifteen questions. (15×2=30)

1) If the product of two right cosets of a subgroup H of a group G is again a right coset of H in G, then prove that H is normal in G.

2) Prove that the subgroup H = {0, 3} of the group $G = (Z_6, +_6)$ is a normal subgroup.

3) If $f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix}, g = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$. Find $f_o g^{-1}$.

4) Show that every homomorphic image of an abelian group is abelian.

5) Define basic feasible solution and degenerate solution of Linear Programming Problem (LPP).

6) Solve 3x + 5y < 15 graphically.

7) Using column minima method determine an initial basic solution of the following transportation problem,

Origin \ Destination	P	Q	R	S	Availability
A	1	2	1	4	30
B	3	3	2	1	50
C	4	2	5	9	20
Requirement	20	40	30	10	100

P.T.O.

8) Define Supremum and Infimum of a sequence.

9) Show that $\left\{\frac{n+1}{n}\right\}$ is a Cauchy sequence.

10) Show that the sequence $\{x_n\}$, whose n^{th} term is $5+\frac{1}{n}$ is monotonic decreasing.

11) Find the limit of the sequence $\left\{\frac{1+4+7+\ldots.+(3n-2)}{2n^2+3n}\right\}$.

12) Define geometric series and discuss its nature.

13) Discuss the nature of the series $\sum_{n=1}^{\infty}(-1)^n \cdot n$.

14) Test the convergence of the series $\sum \frac{n^2}{n!}$.

15) Define conditional convergence and absolute convergence of an alternating series.

16) Sum to infinity of the series $1+\frac{x}{3}+\frac{x^2}{5}+\frac{x^3}{7}+\ldots.$

17) Verify Lagrange's mean value theorem for $f(x) = \log x$ in [1, e].

18) Write the statement of Taylor's theorem.

19) Find the Maclaurin's expansion of Cosx.

20) Evaluate $\lim_{x \to 0} \frac{a^x - b^x}{x}$.

II. Answer **any two** questions. **(2×5=10)**

1) Prove that a subgroup H of a group G is a normal subgroup of G if and only if the product of two right cosets of H in G is also a right coset of H in G.

2) If $f : G \rightarrow G'$ is a homomorphism from the group G into the group G' with Kernel K, then prove that f is one-one iff K = {e}, where 'e' is the identity element of G.

3) If $G = \left\{ \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} \Big/ a \neq 0 \in R \right\}$ is a group w.r. to multiplication of matrices and G' be the group of all nonzero real numbers w.r. to multiplication. Show that $f : G \to G'$ defined by $f\begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} = a$ is an isomorphism. Find its Kernel.

4) Prove that every finite group is isomorphic to a permutation group.

III. Answer any three questions. (3×5=15)

1) Find all the basic solutions for

$x + 2y + z = 4, \quad 2x + y + 5z = 5$

2) Solve the following Lpp graphically,

Maximize $z = 5x + 7y$

subject to constraints
$x + y \leq 4$
$3x + 8y \leq 24$
$10x + 7y \leq 35$
$x, y \geq 0.$

3) Solve the following Lpp by simplex method,

Maximize $z = 3x + 2y$

subject to
$x + y \leq 4$
$x - y \leq 2$
$x, y \geq 0.$

4) Obtain an initial solution for the following transportation problem using Vogel's method.

Distination

	A	B	C	Supply
P	2	7	4	5
Q	3	3	1	8
R	5	4	7	7
S	1	6	2	14
Demand	7	9	18	34

IV. Answer any two questions. (2×5=10)

1) Prove that every convergent sequence has a unique limit point.

2) Discuss the nature of the sequence $\left(x^{\frac{1}{n}}\right)$, $x > 0$ a real number.

3) Find the limit of the 0.5, 0.55, 0.555, ...

V. Answer any four questions. (4×5=20)

1) Let $\sum a_n$ and $\sum b_n$ be two series of positive terms such that

i) $\sum b_n$ is convergent and

ii) $\frac{a_{n+1}}{a_n} \leq \frac{b_{n+1}}{b_n}$ $\forall n$ except for a finite number of terms in the beginning, then prove that $\sum a_n$ is also convergent.

2) State and prove D'Alembert's Ratio test.

3) Examine the convergence of the series $\sum \frac{1}{n^p (n+1)^p}$.

4) Discuss the convergence of the series $\frac{2}{1^2}x + \frac{3^2}{2^3}x^2 + \frac{4^3}{3^4}x^3 + \ldots$

5) Sum to infinity the series $1 + \frac{2}{6} + \frac{2.5}{6.12} + \frac{2.5.8}{6.12.18} + \ldots$

6) If $x > 0$, show that $\frac{x-1}{x+1} + \frac{x^2-1}{2(x+1)^2} + \frac{x^3-1}{3(x+1)^3} + \ldots = \log x$.

VI. Answer any three questions. (3×5=15)

1) Examine the differentiability of the function $f(x) = \begin{cases} x^2 - 1 & \text{for } x \geq 1 \\ 1 - x & \text{for } x < 1 \end{cases}$ at $x = 1$.

2) State and prove Cauchy's mean value theorem.

3) Verify Rolle's theorem for the function $f(x) = x(x+3)\, e^{-\frac{x}{2}}$ in $[-3, 0]$.

4) Expand $\text{Log}_e(1 + e^x)$ in Maclaurin's series upto the term containing x^4.

5) Evaluate $\lim\limits_{x \to 0} (\cos x)^{\cot x}$.